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AN ITERATIVE IMPLEMENTATION FOR 2-D DIGITAL FILTERS.(U)
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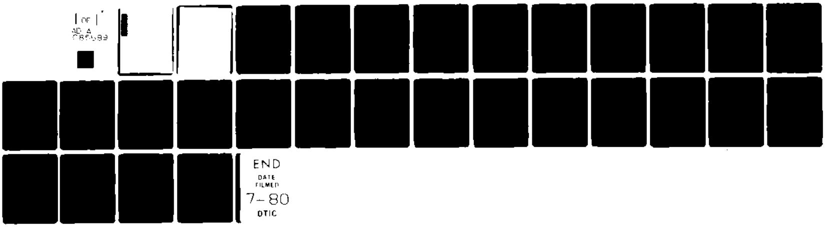

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

AN ITERATIVE IMPLEMENTATION FOR 2-D DIGITAL FILTERS

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LEXINGTON MASSACHUSETTS

ABSTRACT

A 2-D digital filter with a rational frequency response can be expanded into an infinite sequence of filtering operations. Each filtering operation can be implemented by convolution with a low-order 2-D finite-extent impulse response. This sequence of filtering operations can be viewed as an iteration where a new estimate of the output signal is formed from the previous estimate. If a convergence constraint is satisfied, the sequence of estimates will approach the desired output signal. In practice, the number of iterations is finite. Consequently, the frequency response that is actually realized by the iterative implementation is an approximation to the desired rational frequency response.

A

CONTENTS

A. INTRODUCTION

For digital signal processing tasks that are not performed in real time, a processor may have access, at least conceptually, to the entire input signal and the entire output signal. This seems to be particularly true for image processing tasks. An entire image, residing somewhere in the computer system, is used as the input to a processing program that generates an entire output image. In these cases, it seems plausible to consider examining the output image and using it in conjunction with the input image to form a "better" output image.

The iterative implementation for 2-D digital filters described below uses this familiar concept of feedback in an interesting way. A 2-D rational frequency response can be used to form an iterative computation involving only finite-extent impulse response (FIR) filtering operations. The rational frequency response can be theoretically implemented by applying the iterative computation an infinite number of times, providing a convergence criterion is met. In practice, of course, the iterative computation is applied only a finite number of times, so the frequency response that is actually realized is an approximation to the original rational frequency response.

The iterative implementation seems well-suited to digital processors that have the ability to convolve a 2-D signal with a filter kernel of limited extent. With repeated applications of the iterative computation, these processors can be used to implement kernels of much larger extent and greater filtering ability.

Because the iterative computation involves only FIR filtering operations, the iterative implementation can be used to approximate some real symmetric 2-D rational frequency responses without the necessity of 2-D spectral factorization [1,2]. Consequently, 2-D filter design methods that yield real rational frequency responses [1,2] could potentially be used to design filters for the iterative implementation. Furthermore, 2-D real rational frequency responses could be obtained by applying a McClellan-type transformation [3,4] to 1-D real rational frequency responses.

B. DERIVATION OF THE ITERATIVE COMPUTATION

A 2-D rational frequency response can be written

$$H(\omega_1,\omega_2) = A(\omega_1,\omega_2)/B(\omega_1,\omega_2) \tag{1}$$

where $A(\omega_1,\omega_2)$ and $B(\omega_1,\omega_2)$ are trigonometric polynominals given by

$$A(\omega_1,\omega_2) \triangleq \sum_{m_1}\sum_{m_2} a(m_1,m_2)\exp[-j\omega_1 m_1 - j\omega_2 m_2] \tag{2}$$

$$B(\omega_1,\omega_2) \triangleq \sum_{n_1}\sum_{n_2} b(n_1,n_2)\exp[-j\omega_1 n_1 - j\omega_2 n_2] \tag{3}$$

It is assumed that the above sums have a finite number of terms; the arrays $a(m_1,m_2)$ and $b(n_1,n_2)$ have finite extent. Without losing any generality, we can also assume that the ratio $A(\omega_1,\omega_2)/B(\omega_1,\omega_2)$ is normalized so that $b(0,0) \triangleq 1$.

Now we define the function

$$C(\omega_1,\omega_2) \triangleq 1-B(\omega_1,\omega_2)$$

$$= \sum_{n_1}\sum_{n_2} c(n_1,n_2)\exp[-j\omega_1 n_1 - j\omega_2 n_2] \tag{4}$$

where

$$c(n_1,n_2) = -b(n_1,n_2) \quad \text{for } (n_1,n_2)\neq(0,0) \tag{5}$$

$$= 0 \quad \text{for } (n_1,n_2)=(0,0)$$

Using the definition of $C(\omega_1,\omega_2)$, we can write

$$H(\omega_1,\omega_2) = A(\omega_1,\omega_2)/[1-C(\omega_1,\omega_2)] \tag{6}$$

If we define $X(\omega_1,\omega_2)$ as the spectrum of the filter's input signal $x(n_1,n_2)$ and $Y(\omega_1,\omega_2)$ as the spectrum of the filter's output signal $y(n_1,n_2)$ and we assume that both $|Y(\omega_1,\omega_2)|$ and $|X(\omega_1,\omega_2)|$ are bounded, then

$$\begin{aligned} Y(\omega_1,\omega_2) &= H(\omega_1,\omega_2)X(\omega_1,\omega_2) \\ &= A(\omega_1,\omega_2)X(\omega_1,\omega_2)/[1-C(\omega_1,\omega_2)] \end{aligned} \tag{7}$$

or

$$Y(\omega_1,\omega_2) = A(\omega_1,\omega_2)X(\omega_1,\omega_2)+C(\omega_1,\omega_2)Y(\omega_1,\omega_2) \tag{8}$$

This implicit formula for the output spectrum suggests the iterative formulation

$$Y_{-1}(\omega_1,\omega_2) \triangleq 0$$

$$Y_i(\omega_1,\omega_2) \triangleq A(\omega_1,\omega_2)X(\omega_1,\omega_2)+C(\omega_1,\omega_2)Y_{i-1}(\omega_1,\omega_2) \tag{9}$$

After I iterations, we will have

$$Y_I(\omega_1,\omega_2) = A(\omega_1,\omega_2)\ X(\omega_1,\omega_2) \sum_{i=0}^{I} C^i(\omega_1,\omega_2)$$

Letting I goes to infinity, we see that

$$Y_\infty(\omega_1,\omega_2) = A(\omega_1,\omega_2)\ X(\omega_1,\omega_2) \sum_{i=0}^{\infty} C^i(\omega_1,\omega_2)$$

$$= A(\omega_1,\omega_2)\ X(\omega_1,\omega_2)/[1-C(\omega_1,\omega_2)] \qquad (10)$$

The iterative solution will converge to the desired filter output if the geometric series in equation (10) converges. To ensure this, we impose the restriction

$$|C(\omega_1,\omega_2)| < 1 \qquad (11)$$

Because $a(m_1,m_2)$ and $c(n_1,n_2)$ are finite-extent arrays, the filtering operations necessary to implement the iteration (9) are FIR operations. Consequently, the rational frequency response (6) can be implemented exactly by an infinite number of FIR filtering operations.

If condition (11) is indeed true, then it is possible to show that $Y_i(\omega_1,\omega_2)$ converges uniformly to $Y(\omega_1,\omega_2)$ as i goes to infinity. In addition, it is also possible to show that $y_i(n_1,n_2)$, the inverse Fourier transform of $Y_i(\omega_1,\omega_2)$, converges uniformly to $y(n_1,n_2)$ as i goes to infinity.

The use of the iterative computation (9) can be visualized as a simple digital filter that processes a sequence of images (or vectors) rather than a sequence of numbers. Figure 1 shows the iterative computation as a first-order feedback loop, with $x_i(n_1,n_2)$ denoting the input sequence, $y_i(n_1,n_2)$ denoting the output sequence, and i representing the iteration number. The "STORE" operator in Figure 1 stores the result of the previous iteration; it is analogcus to the 1-D shift operator usually represented by z^{-1}. To obtain the output signal $y(n_1,n_2)$ whose Fourier transform $Y(\omega_1,\omega_2)$ is given by equation (7), the input sequence $x_i(n_1,n_2)$ is taken to be a step function in the iteration variable i:

$$\begin{aligned} x_i(n_1,n_2) &= x(n_1,n_2) \quad \text{for } i \geq 0 \\ &= 0 \qquad\qquad\quad \text{for } i < 0 \end{aligned} \tag{12}$$

As suggested by equation (9), the initial condition

$$y_{-1}(n_1,n_2) \triangleq 0 \tag{13}$$

is used to begin the iteration at i=0. Then the desired output $y(n_1,n_2)$ is realized as the steady state output of the filter in Figure 1.

C. STABILITY AND CONVERGENCE

Stability of the filter $H(\omega_1,\omega_2)$ and the satisfaction of the convergence criterion by $C(\omega_1,\omega_2)$ are not equivalent. Consider the filter

$$H(\omega_1,\omega_2) = 1/(1-ae^{-j\omega_1} -ae^{-j\omega_2} + a^2e^{-j\omega_1} e^{-j\omega_2}) \tag{14}$$

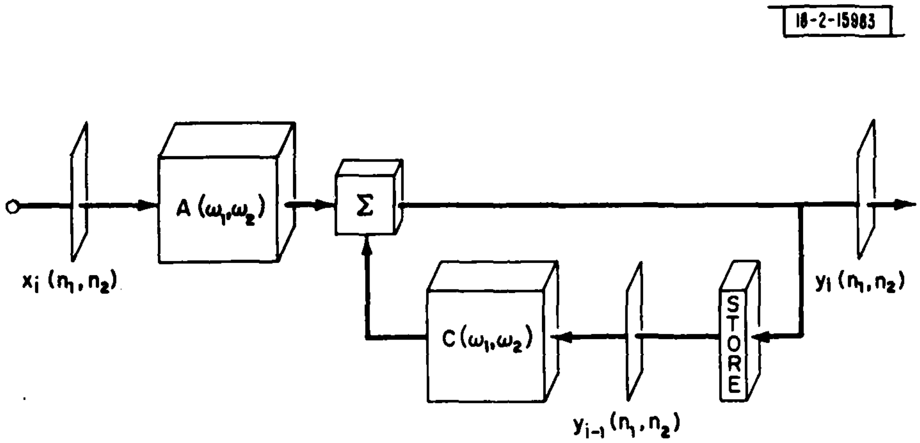

Fig. 1. The iterative computation can be modeled as a first-order feedback loop in the iteration variable i.

Since $H(\omega_1,\omega_2)$ is a separable filter, it is simple to demonstrate that $H(\omega_1,\omega_2)$ is stable if and only if $|a| < 1$. However,

$$C(\omega_1,\omega_2) = ae^{-j\omega_1} + ae^{-j\omega_2} - a^2e^{-j\omega_1} e^{-j\omega_2} \tag{15}$$

so that

$$C(\pi,\pi) = -2a-a^2 \tag{16}$$

In particular, for a=0.5,

$$|C(\pi,\pi)| = 1.25 > 1 \tag{17}$$

which violates the convergence criterion.

In general, if $A(\omega_1,\omega_2)$ is a trigometric polynominal given by equation (2) and $C(\omega_1,\omega_2)$ is a trigometric polynominal given by equations (4) and (5), then $H(\omega_1,\omega_2)$ will be continuous and finite as long as $C(\omega_1,\omega_2) \neq 1$. In this case, the 2-D inverse Fourier transform can be applied to $H(\omega_1,\omega_2)$ to yield the impulse response $h(n_1,n_2)$, which will satisfy

$$\sum_{n_1} \sum_{n_2} |h(n_1,n_2)| < \infty \tag{18}$$

Consequently, the condition $C(\omega_1,\omega_2) \neq 1$ is a sufficient test for BIBO (bounded input-bounded output) stability of $H(\omega_1,\omega_2)$ since we are not concerned with the region of support of the impulse response $h(n_1,n_2)$. Clearly, if the convergence criterion $|C(\omega_1,\omega_2)| < 1$ is satisfied, then $H(\omega_1,\omega_2)$ is stable since $C(\omega_1,\omega_2) \neq 1$. The converse, however, is not true, as the counter-example has shown.

D. TERMINATION OF THE ITERATIVE COMPUTATION

In any practical implementation, the number of iterations actually computed is finite. Using equation (9), it is straightforward to show that the output signal's spectrum after the Ith iteration is given by

$$Y_I(\omega_1,\omega_2) = A(\omega_1,\omega_2)X(\omega_1,\omega_2) \sum_{i=0}^{I} C^i(\omega_1,\omega_2)$$

$$= Y(\omega_1,\omega_2)[1-C^{I+1}(\omega_1,\omega_2)] \qquad (19)$$

We can take the ratio $Y_I(\omega_1,\omega_2)/Y(\omega_1,\omega_2)$ as one measure of the spectral error introduced by terminating the iterative computation after I iterations. The ratio is complex, in general. Thus, for this ratio to lie close to unity, it is necessary to restrict $|C(\omega_1,\omega_2)|$ so that

$$\left| \frac{Y_I(\omega_1,\omega_2)}{Y(\omega_1,\omega_2)} - 1 \right| = \left| C(\omega_1,\omega_2) \right|^{I+1} < \varepsilon \qquad (20)$$

for all frequencies (ω_1,ω_2), where ε is a small, positive constant.

If we specify a tolerable degree of spectral error by fixing ε, we can use this relation to tell us how many iterations will be needed for a given value of $|C(\omega_1,\omega_2)|$. Conversely, it can be used to determine how $|C(\omega_1,\omega_2)|$ must be restricted, as in a filter design algorithm, if the number of iterations I is specified beforehand.

In addition, ε can be used to compute the space domain difference between the output signal at the Ith iteration and the desired output signal. It is straightforward to show that

$$|y(n_1,n_2) - y_I(n_1,n_2)| < \varepsilon Y \tag{21}$$

where

$$Y \triangleq \frac{1}{4\pi^2} \iint_{-\pi}^{\pi} |Y(\omega_1,\omega_2)| d\omega_1 d\omega_2 \tag{22}$$

Although the bound (21) may not be of practical use since Y may not be known beforehand, it nevertheless demonstrates that the sequence of output signals $\{y_i(n_1,n_2)\}$ converges to the desired output signal $y(n_1,n_2)$ when the convergence criterion (11) is satisfied.

The effective frequency response when the iterative computation is terminated after I iterations is given by

$$H_I(\omega_1,\omega_2) = A(\omega_1,\omega_2) \sum_{i=0}^{I} C^i(\omega_1,\omega_2) \tag{23}$$

If we neglect the factor $A(\omega_1,\omega_2)$ for the moment, $H_I(\omega_1,\omega_2)$ has the form of a McClellan transformation filter [4,5] whose tap weights are all equal to one. $H_I(\omega_1,\omega_2)$ can be realized by the structure shown in Figure 2, which embodies the iterative computation as the signal passes from one stage of the structure to the next. Figure 2 serves to illustrate the fact that $H_I(\omega_1,\omega_2)$ represents the frequency response of a 2-D FIR filter. Thus, the filter response actually realized when the iterative computation is terminated is an FIR approximation to the desired rational frequency response $H(\omega_1,\omega_2)$.

In the McClellan transformation, the kernel $C(\omega_1,\omega_2)$ is usually associated with a particular class of 2-D FIR filters such as those exhibiting approximate circular symmetry. The tap weights are varied to realize the mapping of a particular 1-D FIR filter into its 2-D counterpart. However, since the iterative computation has the same form at each iteration, the structure shown in Figure 2 must be identical from one stage to the next, forcing all the tap weights to be equal. The kernel $C(\omega_1,\omega_2)$ itself is varied to approximately realize the desired frequency response $H(\omega_1,\omega_2)$.

Let us define $h_I(n_1,n_2)$ to be the effective 2-D finite-extent impulse response implemented by I iterations of the iterative computation. Because of the nature of the iterative computation, the effective impulse response grows in extent at each iteration. For example, if we assume $A(\omega_1,\omega_2)\overset{\Delta}{=}1$ and $c(n_1,n_2)$ has an NxN region of support, then when the iterative computation is terminated after I iterations, $h_I(n_1,n_2)$ will have an INxIN region of support. The number of multiplications required to implement $h_I(n_1,n_2)$ by the iterative computation however, is only IN^2, in general. Although the area of the nonzero region of $h_I(n_1,n_2)$ goes up proportionally as I^2, the increase in the number of multiplications is proportional to I. Since the terminated iterative implementation can be viewed as a special case of the McClellan transformation implementation, the more detailed results of [5] can be applied to questions of computation and storage.

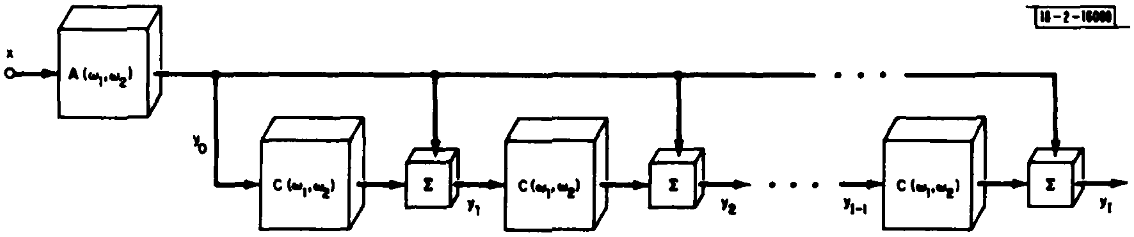

Fig. 2. The iterative computation, when terminated after I iterations, can be represented by McClellan transformation structure with tap weights of unity gain.

E. EFFECTS OF FINITE PRECISION ARITHMETIC

To examine the effects of finite precision arithmetic on the accuracy of the output signal, let us return to the consideration of the iterative computation as the first-order feedback loop shown in Figure 1. The true output spectrum $Y(\omega_1,\omega_2)$ will satisfy the implicit equation

$$Y(\omega_1,\omega_2) = A(\omega_1,\omega_2)X(\omega_1,\omega_2) + C(\omega_1,\omega_2)Y(\omega_1,\omega_2) \tag{24}$$

At each iteration, noise will be injected into the computation. We shall assume that this noise occurs due to fixed-point round-off at the summation box in Figure 1. For a particular ordered pair (n_1,n_2), the convolution of $c(n_1,n_2)$ with $y_{i-1}(n_1,n_2)$, represented by the operator $C(\omega_1,\omega_2)$, and the subsequent addition to the corresponding point of the signal $a(n_1,n_2)*x(n_1,n_2)$ can be performed with double-precision arithmetic and the result rounded back to single precision.

Thus, at each iteration, the computation yields

$$\tilde{Y}_i(\omega_1,\omega_2) = A(\omega_1,\omega_2)X(\omega_1,\omega_2) + C(\omega_1,\omega_2)\tilde{Y}_{i-1}(\omega_1,\omega_2) + \varepsilon_i(\omega_1,\omega_2) \tag{25}$$

where $\tilde{Y}_i(\omega_1,\omega_2)$ is the spectrum of the signal actually computed on the ith iteration and $\varepsilon_i(\omega_1,\omega_2)$ is the spectrum of the computation noise introduced at the ith iteration. If we define

$$E_i(\omega_1,\omega_2) \triangleq Y(\omega_1,\omega_2)-\tilde{Y}_i(\omega_1,\omega_2)$$

and subtract equation (25) from equation (24), we get

$$E_i(\omega_1,\omega_2) = C(\omega_1,\omega_2)E_{i-1}(\omega_1,\omega_2) + \varepsilon_i(\omega_1,\omega_2) \qquad (26)$$

which can be interpreted as a first-order difference equation in the iteration variable i with input $\varepsilon_i(\omega_1,\omega_2)$ and output $E_i(\omega_1,\omega_2)$. Under the assumption that the noise spectrum $\varepsilon_i(\omega_1,\omega_2)$ is uncorrelated from iteration to iteration, the techniques used in chapter 9 of [6] can be applied to equation (26) to give the expression for the output noise power under the assumption that $C(\omega_1,\omega_2)$ satisfies the convergence criterion (11). Thus, if $\sigma_\varepsilon^2(\omega_1,\omega_2)$ represents the noise power at (ω_1,ω_2) due to arithmetic round-off and $\sigma_E{}^2(\omega_1,\omega_2)$ represents the noise power in the output spectrum, then

$$\sigma_E^2\ (\omega_1,\omega_2) = \frac{\sigma_\varepsilon{}^2(\omega_1,\omega_2)}{1 - |C(\omega_1,\omega_2)|^2} \qquad (27)$$

If we further assume that $\sigma_\varepsilon^2\ (\omega_1,\omega_2)$ is a constant and equal to P then the total noise power (TNP) in the output is given by

$$\text{TNP} = \frac{P}{4\pi^2} \iint_{-\pi}^{\pi} \frac{1}{1-|C(\omega_1,\omega_2)|^2}\, d\omega_1 d\omega_2 \qquad (28)$$

Intuitively, we would expect that if $C(\omega_1,\omega_2)$ satisfies the convergence criterion (11), then the effects of the round-off noise $\varepsilon_i(\omega_1,\omega_2)$ introduced at the ith iteration would gradually die out as the iterative computation continues. Of course, additional round-off noise is introduced at each iteration, and equation (28) gives the total noise power in the output signal at the steady state.

If we define $c_i(n_1,n_2)$ as the inverse Fourier transform of $C^i(\omega_1,\omega_2)$, then equation (28) can be re-written using Parseval's theorem.

$$TNP = P \sum_{i=0}^{\infty} \sum_{n_1} \sum_{n_2} c_i^2(n_1,n_2) \qquad (29)$$

When the iterative computation is terminated after I iterations, the summation over i in equation (29) extends only to i=I. This result is similar to that obtained in [5] for the total noise power of a McClellan transformation implementation like Figure 2.

F. A SIMPLE EXAMPLE

Suppose we want to implement a 2-D digital filter whose frequency response is approximately circularly symmetric and is approximately equal to a given real, symmetric 1-D rational function along each frequency axis. The McClellan transformation can be applied to the 1-D rational function to generate a real, symmetric 2-D rational frequency response which can be implemented approximately by the iterative computation. For example, consider the 1-D rational function

$$H(\omega) = \frac{A(\omega)}{1-C(\omega)}$$

where

$$A(\omega) = a(0) + \sum_{m=1}^{4} 2a(m) \cos m\omega$$

$$C(\omega) = \sum_{n=1}^{2} 2c(n) \cos n\omega$$

with

$$a(0) = 0.50033$$
$$2a(1) = 0.81561$$
$$2a(2) = 0.41543$$
$$2a(3) = 0.11311$$
$$2a(4) = -0.000037952$$

and

$$2c(1) = -0.0010153$$
$$2c(2) = -0.83047$$

This function is real and symmetric, and it has the shape shown in Figure 3. When the McClellan transformation for circular symmetry is applied to this function, we get the real, symmetric 2-D rational function shown in Figure 4. This function has the familiar form

$$H(\omega_1,\omega_2) = A(\omega_1,\omega_2)/[1-C(\omega_1,\omega_2)]$$

and furthermore, $C(\omega_1,\omega_2)$ satisfies the convergence criterion (11). Figure 5 gives an indication of the form of $C(\omega_1,\omega_2)$ by showing its value along the ω_1 frequency axis.

After 20 iterations, the surface plot of the effective frequency response of the iterative implementation is not distinguishable from Figure 4 because of the resolution limitations of the plotting device. To observe the convergence of $H_I(\omega_1,\omega_2)$ to the desired $H_I(\omega_1,\omega_2)$, it is more informative to plot the values of $H_I(\omega_1,\omega_2)$ along either of the two frequency axes. Figure 6 shows $H_I(\omega_1,\omega_2)$ evaluated along the ω_1 frequency axis for I=15,20 and 25 iterations.

When comparing Figures 6 and 3, it is easiest to see the difference between $H(\omega_1,\omega_2)$ and $H_I(\omega_1,\omega_2)$ at the origin, where $H(\omega_1,\omega_2)$ and $|C(\omega_1,\omega_2)|$ are large. Here, $|C(0,0)| \cong 0.83$, so that after 15 iterations, the relative error ε defined in equation (20) and expressed as a percentage is approximately 6.1%. After 20 iterations, ε is approximately 2.4%, and after 25 iterations, ε is less than 1%.

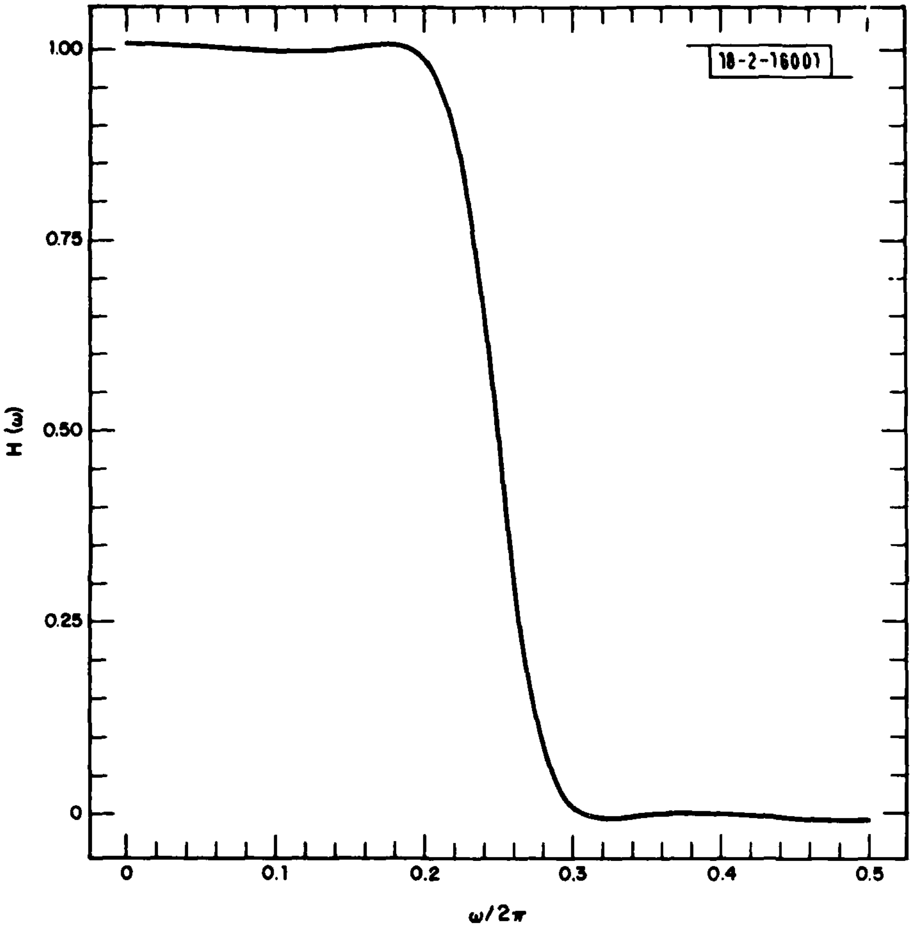

Fig. 3. The real, symmetric 1-D rational frequency response H(ω) is shown for $0 \leq \omega/2\pi \leq 0.5$. It also represents the values of $H(\omega_1,\omega_2)$ shown in Figure 4 along either of the two frequency axes.

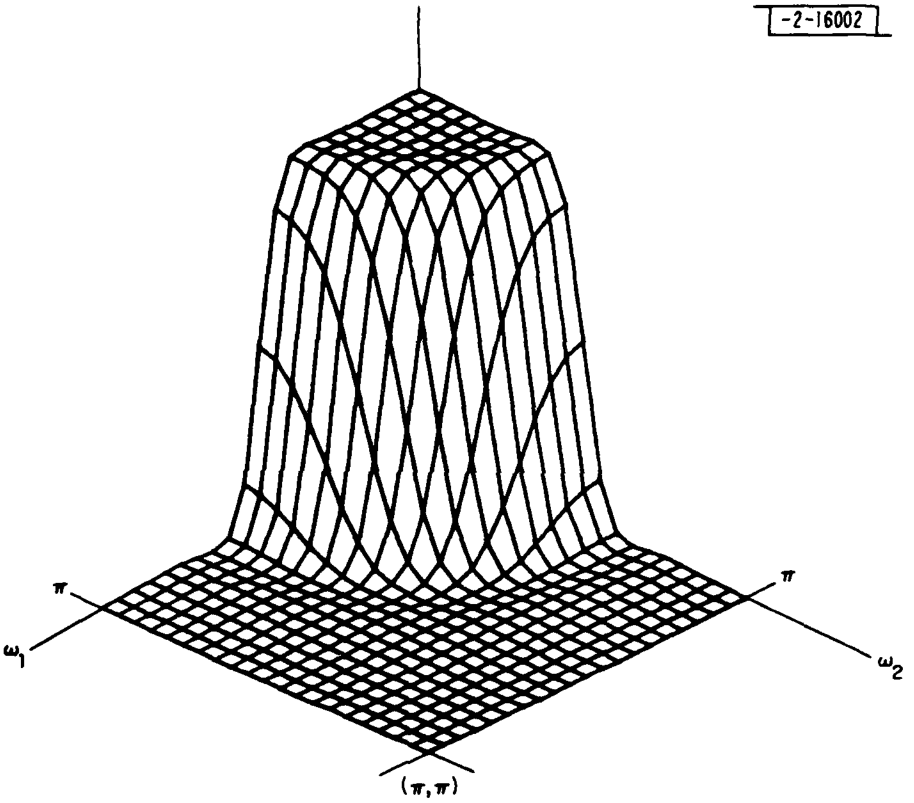

Fig. 4. The real, symmetric 2-D rational frequency response $H(\omega_1,\omega_2)$ is obtained by applying the circularly symmetric McClellan transformation to the function shown in Figure 3. For clarity, only the first quadrant is plotted; the other quadrants are symmetric.

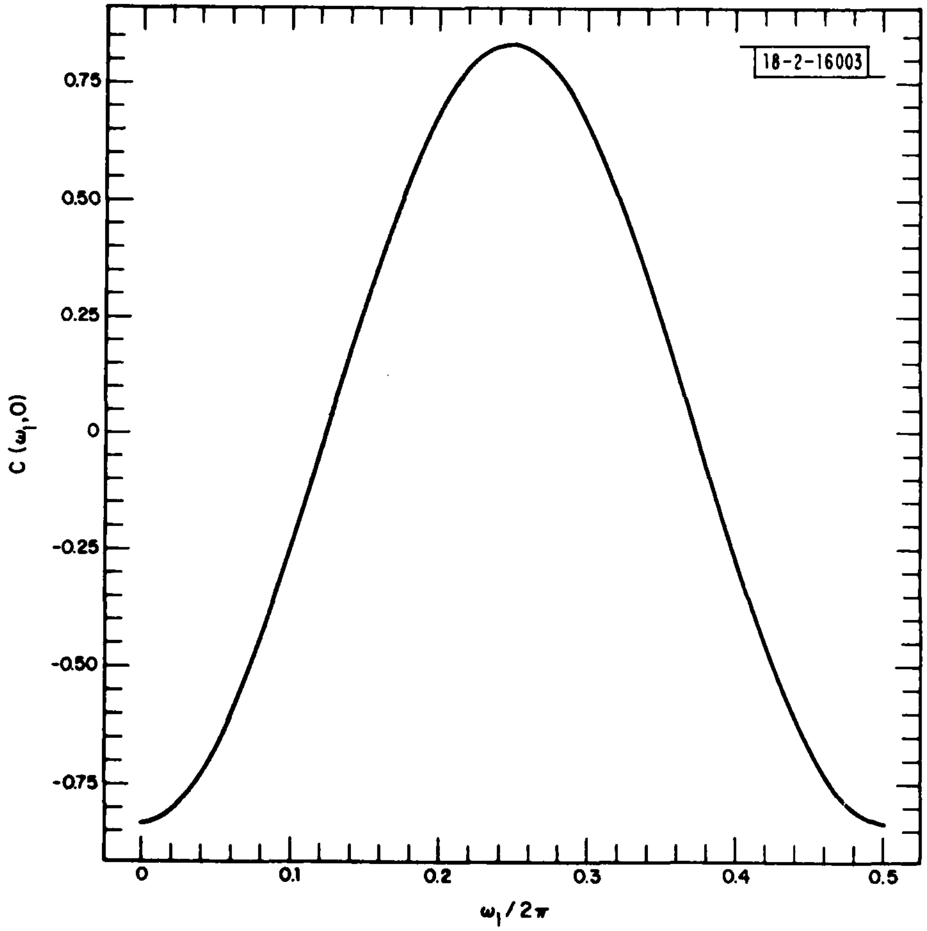

Fig. 5. The function $C(\omega_1,\omega_2)$, evaluated along either of the frequency axes, is equal to $C(\omega)$. It is plotted for $0 \leq \omega/2\pi \leq 0.5$.

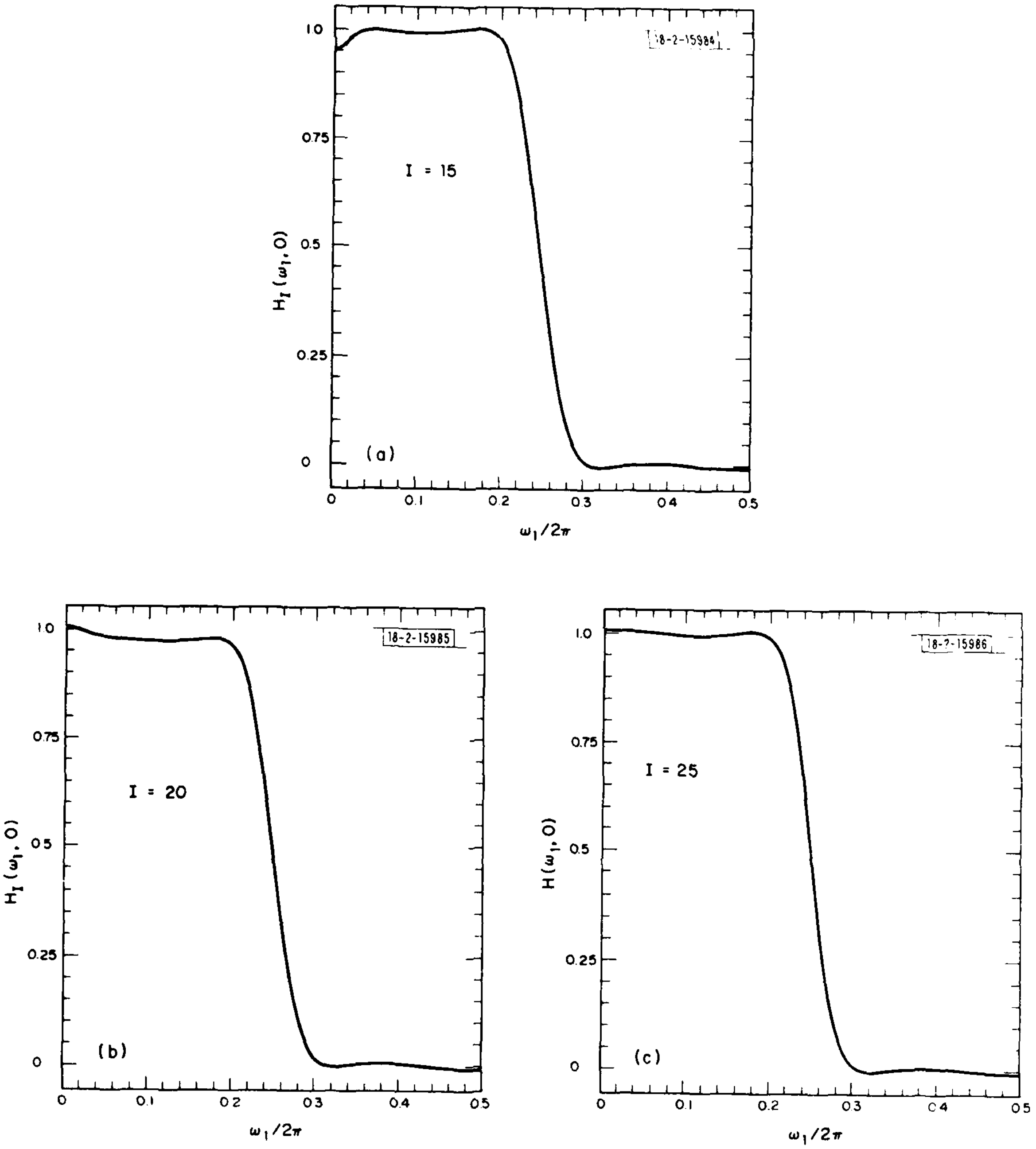

Fig. 6. The frequency $H_I(\omega_1,\omega_2)$ is evaluated along the ω_1 frequency axis for (a). I=15 iterations, (b). I=20 iterations and (c). I=25 iterations. Compare these plots with Figure 3 to note the convergence of $H_I(\omega_1,\omega_2)$ to $H(\omega_1,\omega_2)$.

G. DISCUSSION

The iterative implementation presented here has a number of interesting facets. It can be used to implement a 2-D rational frequency response in an approximate manner by decomposing it into a sequence of FIR filtering operations. Thus, non-causal 2-D IIR filters can be realized approximately by this implementation without performing the computationally-intensive task of spectral factorization (or partial fraction expansion).

The rational frequency response to be implemented does not need to be tested for BIBO stability. Instead, it must satisfy a convergence criterion which is somewhat more restrictive and which implies BIBO stability.

The terminated iterative implementation can be regarded as a special case of a transposed direct form implementation of a McClellan transformation filter (Figure 2), which in turn is a generalization of a transposed direct form tapped delay-line filter [5]. In the terminated iterative implementation, the tap weights all turn out to be equal to one. The structure of figure 2 is constrained to be identical at each stage, since the iterative computation has the same form at each iteration. Because of this property, the terminated iterative implementation does not have quite the generality of the McClellan transformation implementation. On the other hand, the fixed tapped weights do not enter in any design algorithm minimizations, nor do they require an additional multiplication of the 2-D signal by a constant at each iteration.

By viewing the iterative computation as the first-order feedback loop shown in Figure 1, we are led to two possible generalizations of the iterative implementation. The first is to consider more complicated signal flow diagrams such as a second-order feedback loop. The second is to consider an input sequence of 2-D signals which may vary in some way from one iteration to the next.

The iterative implementation provides one method of approximately implementing a real, symmetric 2-D rational transfer function with a device that can realize small-extent FIR filter kernels. It also provides an interesting theoretical bridge between 2-D IIR filters and 2-D FIR filters which may be of use in conceptualizing relationships between these two predominant class of filters.

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A 2-D digital filter with a rational frequency response can be expanded into an infinite sequence of filtering operations. Each filtering operation can be implemented by convolution with a low-order 2-D finite-extent impulse response. This sequence of filtering operations can be viewed as an iteration where a new estimate of the output signal is formed from the previous estimate. If a convergence constraint is satisfied, the sequence of estimates will approach the desired output signal. In practice, the number of iterations is finite. Consequently, the frequency response that is actually realized by the iterative implementation is an approximation to the desired rational frequency response.

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